Homework 02 **Math 261** **Due: Wednesday, Sept 16**

Important: Mathematics is a marathon, not a sprint. Regularly attend class, work on your homework, read your book before class, and ask me questions after class or in office hours.

Please feel comfortable asking me questions. I want to help you succeed!

To do homework: Use your notes and textbook. Read the definitions, theorems, and examples. Look for ``clever ideas" that you can use in the problems you are working on.

To read a math textbook/notes: Use your pencil and some scrap paper! Work on problems as you read them. Try to get the answer *before* you read how it's done.

From Class 03

A. Section 11.3, Exercises # 2 (all parts), 3 (all parts), 5 (omit part c), 6 (all parts), 7

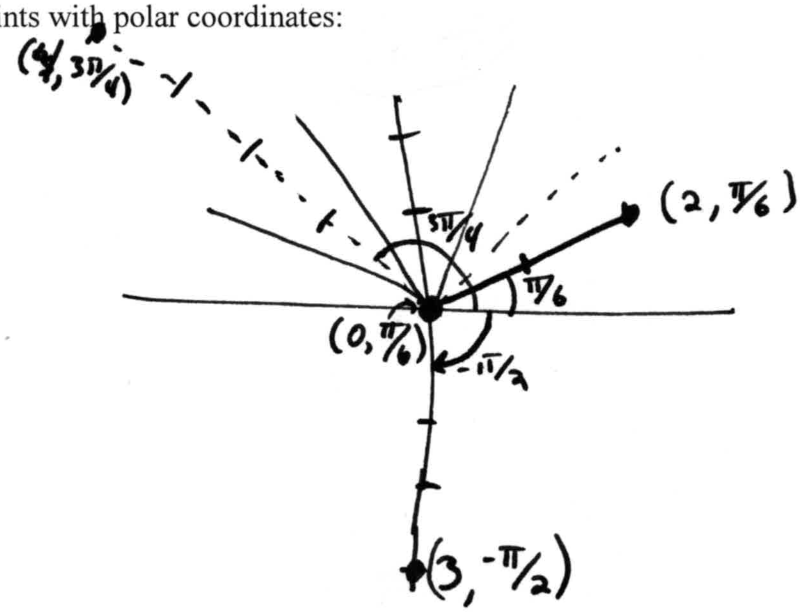
2. Plot the points with polar coordinates:

(a) $(2, \frac{\pi}{6})$

(b) $(4, \frac{3\pi}{4})$

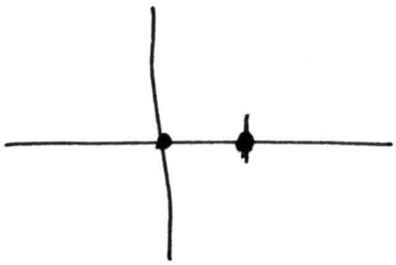
(c) $(3, -\frac{\pi}{2})$

(d) $(0, \frac{\pi}{6})$



3. Convert from rectangular to polar coordinates.

(a) (1, 0)



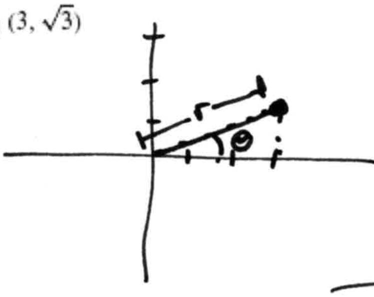
By observation

$\theta = 0$

$r = 1$

$(1, 0)$

(b) $(3, \sqrt{3})$



know:

$r = \sqrt{3^2 + (\sqrt{3})^2}$

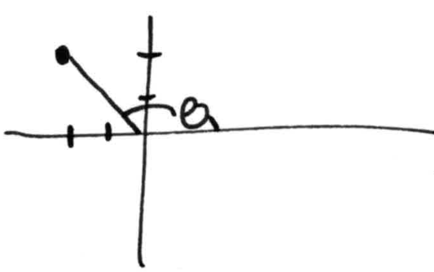
$= \sqrt{9 + 3}$

$= \sqrt{12}$

know $\tan(\theta) = \frac{y}{x} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ & know $\tan(\pi/6) = \frac{1}{\sqrt{3}}$

Choose: $\theta = \pi/6$ $\Rightarrow$ $(\sqrt{12}, \pi/6)$

(c) (−2, 2)



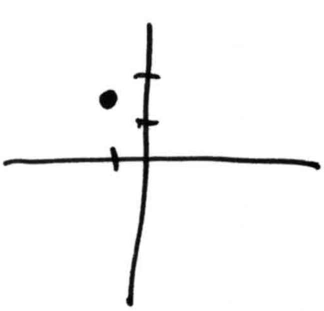
$r = \sqrt{(-2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8}$

$\tan(\theta) = \frac{y}{x} = \frac{2}{-2} = -1$

know: $\tan(\pi/4) = 1$ & θ in quadrant II

Choose: $\theta = 3\pi/4$ $\Rightarrow$ $(\sqrt{8}, 3\pi/4)$

(d) $(-1, \sqrt{3})$



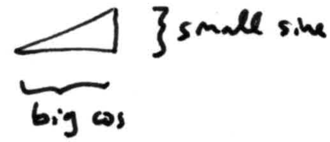
$r = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\tan(\theta) = \frac{y}{x} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

know $\tan(\pi/3) = \sqrt{3}$ & θ in quadrant II

$\Rightarrow$ choose $\theta = 2\pi/3$

So $(2, 2\pi/3)$



5. Convert from polar to rectangular coordinates:

(a) $(3, \frac{\pi}{6})$

$$\begin{cases} x = r \cdot \cos\theta \\ y = r \cdot \sin\theta \end{cases}$$

$$x = 3 \cdot \cos(\pi/6) = 3 \cdot \frac{\sqrt{3}}{2}$$

$$y = 3 \cdot \sin(\pi/6) = 3 \cdot \frac{1}{2}$$

$$\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$$

(b) $(6, \frac{3\pi}{4})$

$$x = 6 \cdot \cos(3\pi/4) = 6 \cdot \frac{-\sqrt{2}}{2} = -3\sqrt{2}$$

$$y = 6 \cdot \sin(3\pi/4) = 6 \cdot \frac{\sqrt{2}}{2} = 3\sqrt{2}$$

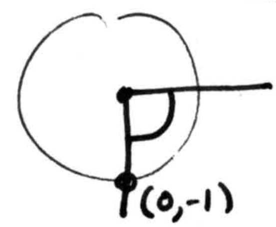
sine pos

$$(-3\sqrt{2}, 3\sqrt{2})$$

(d) $(5, -\frac{\pi}{2})$

$$x = 5 \cdot \cos(-\pi/2) = 5 \cdot 0 = 0$$

$$y = 5 \cdot \sin(-\pi/2) = 5 \cdot (-1) = -5$$



$$(0, -5)$$

(0, -2)

6. Which of the following are possible polar coordinates for the point P with rectangular coordinates $(0, -2)$?

✗ (a) $\left(2, \frac{\pi}{2}\right)$

✓ **(b)** $\left(2, \frac{7\pi}{2}\right)$

only (b) and (c)

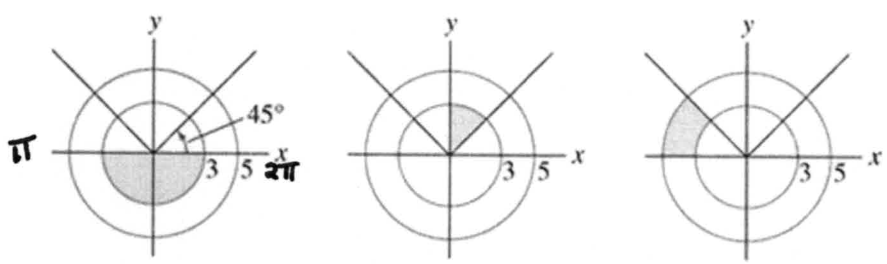
✓ **(c)** $\left(-2, -\frac{3\pi}{2}\right)$

✗ **(d)** $\left(-2, \frac{7\pi}{2}\right)$

✗ **(e)** $\left(-2, -\frac{\pi}{2}\right)$

✗ **(f)** $\left(2, -\frac{7\pi}{2}\right)$

7. Describe each shaded sector in Figure 17 by inequalities in r and θ.



(A) (B) (C)

(A) $\pi \le \theta \le 2\pi$ and $0 \le r \le 3$

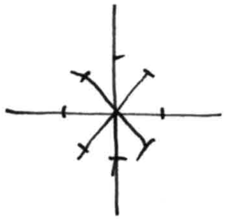
(B) $\pi/4 \le \theta \le \pi/2$ and $0 \le r \le 3$

(C) $3\pi/4 \le \theta \le \pi$ and $3 \le r \le 5$

B. Section 11.3, Exercises 11,12,14,15,17,19,20, 27,28,29,31

In Exercises 11–16, convert to an equation in rectangular coordinates.

11. $r = 7$



defines circle of radius 7 around (0,0)

⇒ know cartesian eqn is

$$x^2+y^2=7^2$$

12. $r = \sin\theta$

Rewrite

$$r^2 = r\cdot\sin\theta$$

Substitute

$$\boxed{x^2+y^2=x}$$

want to "find"

$r^2 = x^2+y^2$

$x = r\cdot\cos\theta$

$y = r\cdot\sin\theta$

---

14. $r = 2\csc\theta$

$$r = 2\cdot\frac{1}{\cos\theta}$$

$$r\cdot\cos\theta = 2$$

$$\boxed{x=2}$$

want to find

$r^2 = x^2+y^2$

$x = r\cdot\cos\theta$

$y = r\cdot\sin\theta$

---

15. $r = \frac{1}{\cos\theta - \sin\theta}$

$$r(\cos\theta - \sin\theta) = 1$$

$$r\cdot\cos\theta - r\cdot\sin\theta = 1$$

$$x - y = 1$$

want to find

$r^2 = x^2+y^2$

$x = r\cdot\cos\theta$

$y = r\cdot\sin\theta$

In Exercises 17–20, convert to an equation in polar coordinates.

<u>17.</u> $x^2 + y^2 = 5$

$x = r\cdot\cos\theta \,,\quad y = r\cdot\sin\theta$

plug in

$(r\cdot\cos\theta)^2 + (r\cdot\sin\theta)^2 = 5$

$r^2\cdot\cos^2\theta + r^2\cdot\sin^2\theta = 5$

solve for r

$r^2(\cos^2\theta + \sin^2\theta) = 5$

$r^2 = \dfrac{5}{\cos^2\theta + \sin^2\theta}$

<u>19.</u> $y = x^2$

$r\cdot\sin\theta = (r\cdot\cos\theta)^2$

$r\cdot\sin\theta = r^2\cos\theta$

$\dfrac{\sin\theta}{\cos\theta} = r$

$r = \tan(\theta)$

<u>20.</u> $xy = 1$

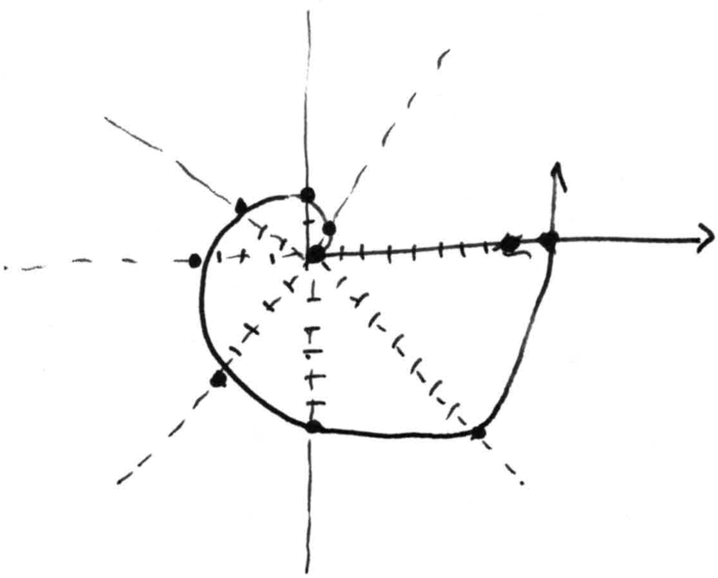
$(r\cdot\sin\theta)\cdot(r\cdot\cos\theta) = 1$

$r^2\cdot\sin\theta\cdot\cos\theta = 1$

$r^2 = \dfrac{1}{\sin\theta\cdot\cos\theta}$

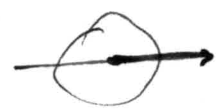
27. Sketch the curve $r = \frac{1}{2}\theta$ (the spiral of Archimedes) for between 0 and 2 π by plotting the points for $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}, \ldots, 2\pi$.

θ	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	...
r	0	$\pi/8$	$\pi/4$	$3\pi/8$	$\pi/2$	...

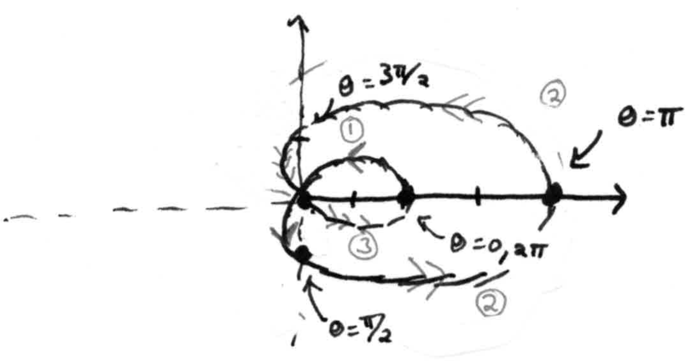


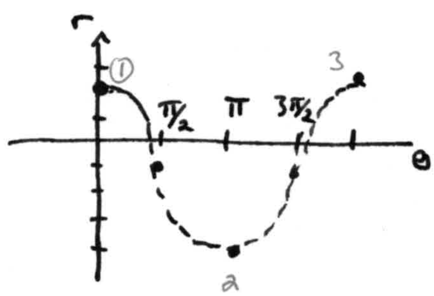
each tick mark ≈ one $\pi/8$

28. Sketch $r = 3\cos\theta - 1$ by plotting when $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ *and* by using the graph of r with respect to θ (see also Example 8).



θ	0	$\pi/2$	π	$3\pi/2$	2π
r	$3\cdot\cos(0)-1 = 3-1 = 2$	$3\cdot\cos(\pi/2)-1 = 3\cdot 0-1 = 0-1$	$3\cdot\cos(\pi)-1 = 3(-1)-1 = -4$	$3\cdot\cos(3\pi/2)-1 = 3\cdot 0-1 = -1$	$3\cdot\cos(2\pi)-1 = 3\cdot 1-1 = 2$

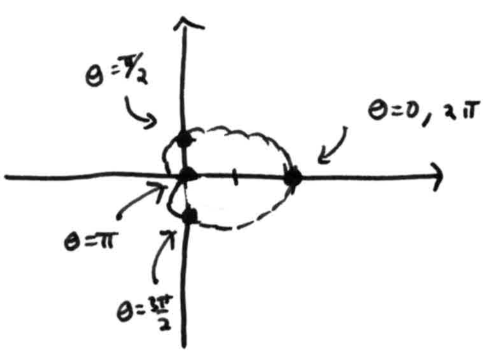




29. Sketch the cardioid curve $r = 1 + \cos\theta$ using $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ *and* by using the graph of r with respect to θ

θ	0	$\pi/2$	π	$3\pi/2$	2π
r	$1+\cos(0) = 1+1 = 2$	$1+\cos(\pi/2) = 1+0 = 1$	$1+\cos(\pi) = 1+(-1) = 0$	$1+\cos(\frac{3\pi}{2}) = 1+0 = 1$	$1+\cos(2\pi) = 1+1 = 2$

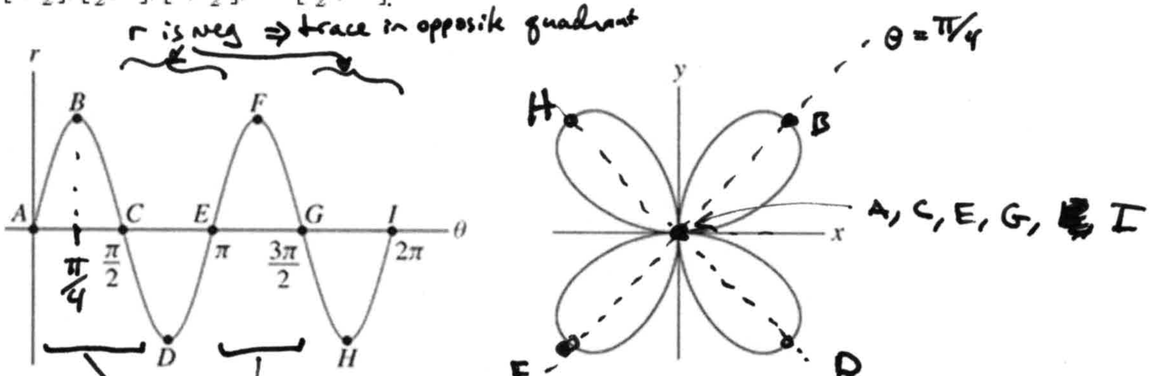
$r = 1 + \cos\theta$



31. Figure 20 displays the graphs of $r = \sin 2\theta$ in rectangular coordinates and in polar coordinates, where it is a "rose with four petals." Identify:

(a) The points in (B) corresponding to points A–I in (A).

(b) The parts of the curve in (B) corresponding to the angle intervals $[0, \frac{\pi}{2}]$, $[\frac{\pi}{2}, \pi]$, $[\pi, \frac{3\pi}{2}]$, and $[\frac{3\pi}{2}, 2\pi]$.



(A) Graph of r as a function of θ, where $r = \sin 2\theta$.

(B) Graph of $r = \sin 2\theta$ in polar coordinates.

Figure 20

r is pos

[redacted] ⇒ trace in this quadrant

$r = 0$ when $\theta = \pi/2, \pi, 3\pi/2, 2\pi$

$[0, \pi/2]$:

$[\pi/2, \pi]$:

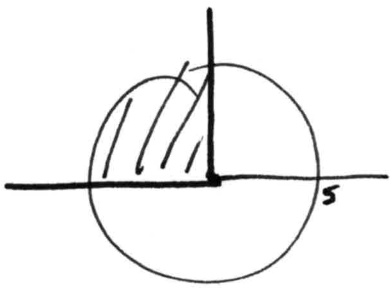
$[\pi, 3\pi/2]$:

$[3\pi/2, 2\pi]$

From Class 04

C. Section 11.4, Exercises 1,3,5,6,7,8,9 (just compute area), 11,12,13,14,15,16

1. Sketch the area bounded by the circle $r = 5$ and the rays $\theta = \frac{\pi}{2}$ and $\theta = \pi$, and compute its area as an integral in polar coordinates.



$$\int_{\text{start }\theta}^{\text{end }\theta} \frac{1}{2}\cdot r^2 \, d\theta = \int_{\pi/2}^{\pi} \frac{1}{2} 5^2 \, d\theta$$

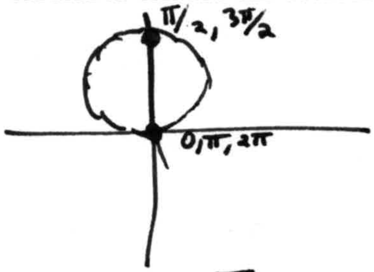
$$= \int_{\pi/2}^{\pi} \frac{25}{2} \, d\theta = \frac{25}{2}\theta \Big]_{\pi/2}^{\pi}$$

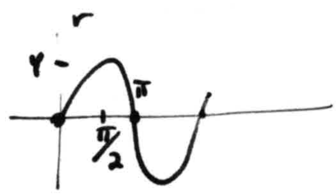
$$= \frac{25}{2}\cdot\frac{2\pi}{2} - \frac{25}{2}\cdot\frac{\pi}{2}$$

$$= \frac{25\pi}{4}$$

this checks out! it is 1/4 of the area of the whole circle

3. Calculate the area of the circle $r = 4 \sin \theta$ as an integral in polar coordinates (see Figure 4). Be careful to choose the correct limits of integration.





$4^2 = 16$

$4^2 \cdot \frac{1}{2} = 8$

area of ONE circle $= \int_0^{\pi} \frac{1}{2} r^2 \, d\theta = \int_0^{\pi} \frac{1}{2}(4\cdot\sin\theta)^2 \, d\theta$

$$= \int_0^{\pi} 8\cdot\sin^2\theta \, d\theta = \int_0^{\pi} 4 \cdot (1-\cos 2\theta) \, d\theta$$

$$= \int_0^{\pi} 4 \, d\theta - \int_0^{\pi} 4\cdot\cos 2\theta \, d\theta$$

$$= [4\theta]_0^{\pi} - \left[2\sin(2\theta)\right]_0^{\pi} = (4\pi - 0) - (2\cdot\sin(2\pi) - 2\cdot\sin(0))$$

$$= 4\pi$$

Scrap:

$(\theta^2+4\theta)^2$
$= \theta^2\cdot\theta^2 + 2\cdot 4\theta\cdot\theta^2 + 4\theta\cdot 4\theta$
$= \theta^4 + 8\theta^3 + 16\theta^2$

$4\cdot 2 = 16\cdot 2 = 32$

5. Find the area of the shaded region in Figure 14. Note that θ varies from 0 to $\frac{\pi}{2}$.

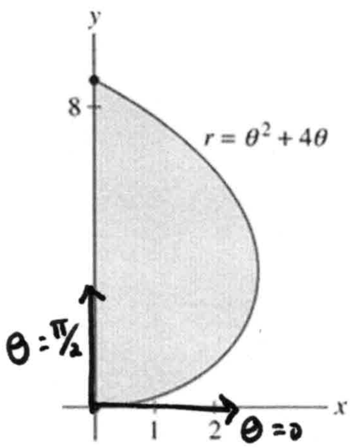


Figure 14

$$\int_0^{\pi/2} \frac{1}{2} r^2 \, d\theta = \int_0^{\pi/2} \frac{1}{2}(\theta^2+4\theta)^2 \, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2}(\theta^4 + 8\theta^3 + 16\theta^2) \, d\theta$$

$$= \int_0^{\pi/2} \frac{\theta^4}{2} + 4\theta^3 + 8\theta^2 \, d\theta$$

$$= \left[\frac{\theta^5}{10} + \frac{4\theta^4}{4} + \frac{8\theta^3}{3}\right]_0^{\pi/2}$$

$$= \frac{(\pi/2)^5}{10} + \left(\frac{\pi}{2}\right)^4 + \frac{8}{3}\cdot\left(\frac{\pi}{2}\right)^3$$

6. Which interval of θ-values corresponds to the the shaded region in Figure 15? Find the area of the region.

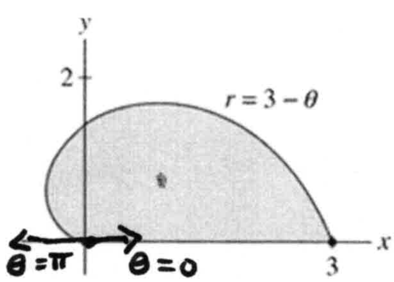


Figure 15

$$\int_0^{\pi} \frac{1}{2}(3-\theta)^2 \, d\theta$$

$$= \int_0^{\pi} \frac{1}{2}(9 - 6\theta + \theta^2) \, d\theta$$

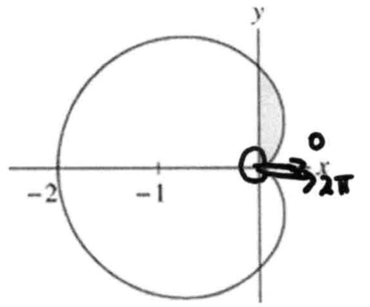
$$= \int_0^{\pi} \frac{9}{2} - 3\theta + \frac{1}{2}\theta^2 \, d\theta$$

$$= \left[\frac{9}{2}\theta - \frac{3}{2}\theta^2 + \frac{1}{2}\cdot\frac{\theta^3}{3}\right]_0^{\pi}$$

$$= \frac{9}{2}\pi - \frac{3}{2}\pi^2 + \frac{1}{6}\pi^3$$

$r = 1 - \cos\theta$

7. Find the total area enclosed by the cardioid in Figure 16.



$$\int_0^{2\pi} \frac{1}{2}(1-\cos\theta)^2 \, d\theta$$

$$= \int_0^{2\pi} \frac{1}{2}\left(1 - 2\cos(\theta) + \cos^2\theta\right) d\theta$$

$$= \int_0^{2\pi} \frac{1}{2} \, d\theta - \int_0^{2\pi} \cos\theta \, d\theta + \int \frac{1}{2}\cos^2\theta \, d\theta$$

$$= \left[\frac{1}{2}\theta\right]_0^{2\pi} - \left[\sin(\theta)\right]_0^{2\pi} + \int_0^{2\pi} \frac{1}{2} \cdot \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= (\pi - 0) - (0 - 0) + \int_0^{2\pi} \left(\frac{1}{4} + \frac{1}{4}\cos 2\theta\right) d\theta$$

$$= \pi + \left[\frac{1}{4}\theta + \frac{1}{4}\frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \pi + \left((\pi/2 + 0) - (0 + 0)\right) = \frac{3\pi}{2}$$

8. Find the area of the shaded region in Figure 16.

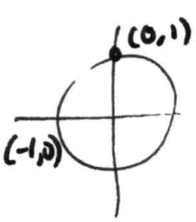
area of shaded region

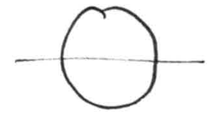
$$= \left[\frac{1}{2}\theta - \sin(\theta) + \frac{1}{4}\theta + \frac{1}{8}\sin(2\theta)\right]_0^{\pi/2}$$

$$= \left(\left(\frac{\pi}{4} - \sin\left(\frac{\pi}{2}\right) + \frac{\pi}{8} + \frac{1}{8}\sin(\pi)\right) - (0 - 0 + 0 + 0)\right)$$

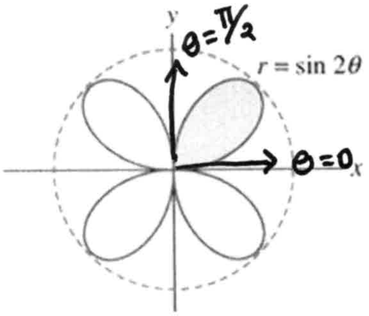
$$= \pi/4 - 1 + \frac{\pi}{8} + 0$$

$$= \frac{3\pi}{8} - 1$$





9. Find the area of one leaf of the "four-petaled rose" $r = \sin 2\theta$ (Figure 17).



check: $r = 0 \iff \sin(2\theta) = 0$

$\iff (2\theta) = 0, \pi, \ldots$

$\iff \theta = 0, \pi/2, \ldots$

✓

$$\text{area of one leaf} = \int_0^{\pi/2} \frac{1}{2} r^2 \, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2} (\sin(2\theta))^2 \, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{2} \left(\frac{1 - \cos(4\theta)}{2} \right) d\theta$$

know

$$\sin^2\theta = \frac{1 - \cos(2\theta)}{2}$$

so

$$\sin^2(2\theta) = \frac{1 - \cos(4\theta)}{2}$$

$$= \frac{1}{4} \cdot \int_0^{\pi/2} 1 - \cos(4\theta) \, d\theta$$

← either do u-sub or think $\frac{d}{d\theta}\left[\frac{\sin(4\theta)}{4}\right] = \frac{\cos 4\theta \cdot 4}{4}$

$$= \frac{1}{4} \left[\theta - \frac{\sin(4\theta)}{4} \right]_0^{\pi/2}$$

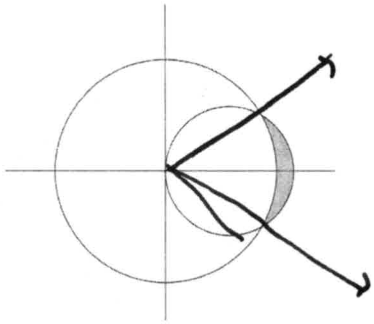
$$= \frac{1}{4} \left(\frac{\pi}{2} - \frac{\sin(4 \pi/2)}{4} \right) - \frac{1}{4} \left(0 - \frac{\sin(0)}{4} \right)$$

$\sin(2\pi) = 0$

$\sin(0) = 0$

$$= \frac{\pi}{8}$$

14A. Find the area of the region outside the circle $r = \sqrt{3}$ and inside the curve $r = 2\cos(\theta)$.



intersect when

$$\sqrt{3} = 2 \cdot \cos\theta$$

when

$$\frac{\sqrt{3}}{2} = \cos\theta$$

when

$$\theta = \pi/6,\ -\pi/6,\ \ldots$$

Shaded area = area under outside − area under inside

$$= \int_{-\pi/6}^{\pi/6} \frac{1}{2}(2\cos\theta)^2\,d\theta - \int_{-\pi/6}^{\pi/6} \frac{1}{2}(\sqrt{3})^2\,d\theta$$

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

$$= \int_{-\pi/6}^{\pi/6} \frac{1}{2}\cdot 4\cdot\cos^2\theta\,d\theta - \int_{-\pi/6}^{\pi/6} \frac{1}{2}3\,d\theta$$

$$= \int_{-\pi/6}^{\pi/6} 2\left(\frac{1+\cos(2\theta)}{2}\right)d\theta - \left[\frac{3\theta}{2}\right]_{-\pi/6}^{\pi/6}$$

u-sub or

$$\frac{d}{d\theta}\left[\frac{\text{[redacted]}n(2\theta)}{2}\right] = \frac{\cos(2\theta)\cdot 2}{2}$$

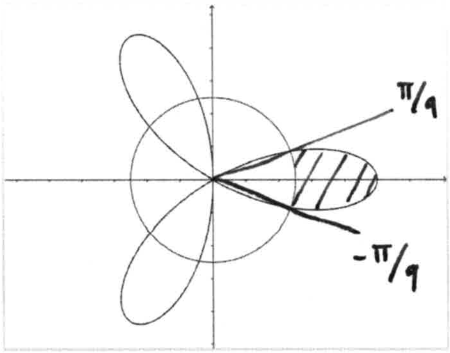
$$= \left[\theta + \left(\frac{\sin(2\theta)}{2}\right)\right]_{-\pi/6}^{\pi/6} - \left(\frac{3\pi}{2\cdot 6} + \frac{3\pi}{2\cdot 6}\right) \quad (\pi/4,\ \pi/4)$$

$$\sin(\pi/3) = \frac{\sqrt{3}}{2} \qquad \sin(-\pi/3) = -\frac{\sqrt{3}}{2}$$

$$= \left(\frac{\pi}{6} - \frac{\sin(2\cdot\pi/6)}{2}\right) - \left(-\frac{\pi}{6} + \frac{\sin(2\cdot -\pi/6)}{2}\right) - \frac{\pi}{2}$$

$$= \pi/6 - \frac{\frac{\sqrt{3}}{2}}{2} + \frac{\pi}{6} + \frac{(-)-\frac{\sqrt{3}}{2}}{2} - \frac{\pi}{2} = \boxed{\frac{\pi}{3} + \frac{\sqrt{3}}{2} - \pi} = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$$

14B. Find the area inside one leaf of the rose $r = \cos(3\theta)$ which is outside of the circle $r = \frac{1}{2}$.



curves intersect when

$$\frac{1}{2} = \cos(3\theta)$$

when

$$3\theta = \frac{-\pi}{3}, \frac{\pi}{3}, \dots$$

$$\theta = \frac{-\pi}{9}, \frac{\pi}{9}, \dots$$

Shaded area = area under outer − area under inner

$$= \int_{-\pi/9}^{\pi/9} \frac{1}{2}(\cos(3\theta))^2\, d\theta - \int_{-\pi/9}^{\pi/9} \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2 d\theta$$

Remember

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2}$$

so

$$\cos^2(3\theta) = \frac{1+\cos(6\theta)}{2}$$

$$= \int_{-\pi/9}^{\pi/9} \frac{1}{2}\left(\frac{1+\cos(6\theta)}{2}\right) d\theta - \int_{-\pi/9}^{\pi/9} \frac{1}{8}\, d\theta$$

$$= \frac{1}{4}\cdot\int_{-\pi/9}^{\pi/9} 1+\cos(6\theta)\, d\theta - \left[\frac{\theta}{8}\right]_{-\pi/9}^{\pi/9}$$

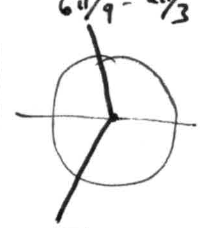
$$= \frac{1}{4}\cdot\left[\theta + \frac{-\sin(6\theta)}{6}\right]_{-\pi/9}^{\pi/9} - \left(\frac{\frac{\pi}{9}}{8} + \frac{\frac{\pi}{9}}{8}\right)$$

Aside:

$$\left(\frac{\pi}{9\cdot 8}\cdot 2 = \frac{\pi}{4\cdot 9}\right)$$

$$= \frac{1}{4}\left(\frac{\pi}{9} - \frac{\sin\left(\frac{6\pi}{9}\right)}{6}\right) - \frac{1}{4}\left(-\frac{\pi}{9} + \frac{\sin\left(\frac{-6\pi}{9}\right)}{6}\right) - \left(\frac{\pi}{4\cdot 9}\right)$$

($\frac{6\pi}{9} = \frac{2\pi}{3}$, $\frac{-6\pi}{9} = -\frac{2\pi}{3}$)



$$= \frac{1}{4}\cdot\frac{\pi}{9} - \frac{1}{4}\cdot\frac{\frac{\sqrt{3}}{2}}{6} + \frac{1}{4}\cdot\frac{\pi}{9} + \frac{1}{4}\cdot\frac{-\frac{\sqrt{3}}{2}}{6} - \frac{\pi}{4\cdot 9}$$

= ...

15. Find the area of the inner loop of the limaçon with polar equation $= 2\cos\theta - 1$.

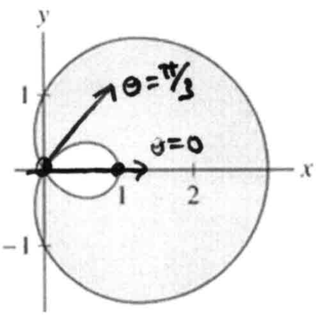
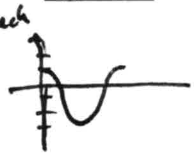


Figure 21 The limaçon $r = 2\cos\vartheta - 1$.

check



inner loop begins/ends when

$$0 = 2\cdot\cos\theta - 1$$

$$\Leftrightarrow$$

$$1 = 2\cdot\cos\theta$$

$$\cos\theta = 1/2$$

$$\theta = \pi/3,\ -\pi/3,\ \dots$$

Δ

$$\text{half of inner loop} = \int_0^{\pi/3} \tfrac{1}{2}\cdot r^2\, d\theta$$

$$= \int_0^{\pi/3} \tfrac{1}{2}\left(2\cdot\cos(\theta) - 1\right)^2 d\theta$$

$$= \int_0^{\pi/3} \tfrac{1}{2}\left(4\cos^2\theta - 4\cdot\cos\theta + 1\right) d\theta$$

$$= \int_0^{\pi/3} 2\cdot\cos^2\theta - 2\cos\theta + 1\ d\theta$$

Recall $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$$= \int_0^{\pi/3} \cancel{2}\cdot\left(\frac{1+\cos(2\theta)}{\cancel{2}}\right) - 2\cdot\cos\theta + 1\ d\theta$$

$\hookrightarrow 1 + \cos(2\theta)$

u-sub on thing

$$\frac{d}{d\theta}\left[\frac{-\sin 2\theta}{2}\right] = \frac{\cos(2\theta)\cdot 2}{2}$$

$$= \left[\theta + \frac{-\sin 2\theta}{2} - 2\cdot(-\sin\theta) + \theta\right]_0^{\pi/3}$$

$$= \left(\frac{\pi}{3} - \frac{\sin(\frac{2\pi}{3})}{2} + 2\cdot\sin\left(\frac{\pi}{3}\right) + \frac{\pi}{3}\right) - (0+0+0+0)$$

$$\text{half of inner loop} = \frac{2\pi}{3} - \frac{1}{2}\cdot\frac{\sqrt{3}}{2} + 2\cdot\frac{\sqrt{3}}{2} = \frac{2\pi}{3} + \frac{3\sqrt{3}}{4}$$

$$-\frac{\sqrt{3}}{4} + \frac{4\sqrt{3}}{4} = \frac{3\sqrt{3}}{4}$$

$$\boxed{\text{whole inner loop} = \frac{4\pi}{3} + \frac{3\sqrt{3}}{2}}$$

16. Find the area of the shaded region in Figure 21 between the inner and outer loop of the limaçon $r = 2\cos\theta - 1$.

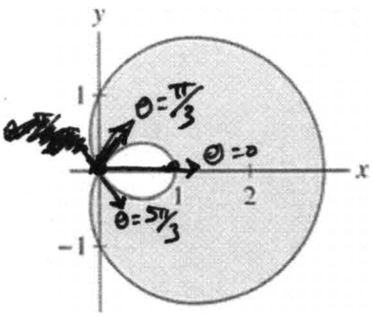


Figure 21 The limaçon $r = 2\cos\theta - 1$.

trace whole outer loop between $\theta = \frac{\pi}{3}$ and $\theta = \frac{5\pi}{3}$

Previous eg

area between = area under outer − area under inner

$$= \int_{\pi/3}^{5\pi/3} \frac{1}{2}(2\cdot\cos\theta - 1)^2 d\theta - 2\cdot\int_0^{\pi/3} \frac{1}{2}(2\cdot\cos - 1)^2 d\theta$$

one whole outer loop

two half inner loops